Seminar on Public Finance

Lecture #3: January 30

Economic Efficiency

Measuring efficiency

- We will the measure efficiency costs of taxation by the amount of deadweight loss (DWL)
- Sometime we'll call this the "excess burden" of taxation since it's the amount, in excess of revenue collected, that the producers and consumers would need to be compensated in order to be as well off as before the tax.
- Our favorite shape here will be the triangle.

What is deadweight loss? (1)

- Thus far, we have focused on the incidence of government policies
 - i.e., how price interventions affect equilibrium prices and factor returns.
 - How taxes affect the slices of the pie.
- A second question is how taxes affect the size of the pie.
- Example: income taxation:
 - Government raises taxes:
 - To raise revenue for the purchase of public goods (e.g. tanks)
 - To redistribute income from rich to poor.
 - But raising tax revenue generally has an efficiency cost: to generate \$1 in revenue, need to reduce welfare of taxed individuals by more than \$1
 - Efficiency costs come from distortions in behavior.

What is deadweight loss? (2)

- There is a large set of studies on how to implement policies that minimize efficiency costs (optimal taxation).
- This is the core theory of public finance and is adapted to study transfer programs, etc.
- We begin with positive analysis of how to measure efficiency cost ("excess burden" (EB) or "deadweight cost") of a given tax system.
 - Computing EB gives you the cost of taxation (often referred to as the marginal cost of public funds).
 - We will see that this number is not uniquely defined
- Note: EB does not tell you anything about the benefit of taxation (redistribution, raise money for public goods,...).
 - Ultimately we will weigh DWL and the benefits of what is done with the taxes raised

Marshallian Surplus and the Harberger Formula/Triangle (1)

- Start with the simplest case
- Two good model with representative consumer and firm
 - x=taxed good, y=untaxed good, p=producer price of x (before tax), τ=tax on x, Z=income
- Key assumptions: quasilinear utility (<u>no income effects</u>), competitive production
- No income effect means Marshallian can give us the welfare effects; can examine in simple supply and demand setting
- Consumer solves:

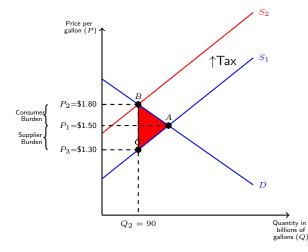
$$\max_{x,y} u(x) + y, \quad s.t. \quad (p+\tau)x + y = Z$$
(1)

Marshallian Surplus (2)

- Price taking firms use c(S) units of \boldsymbol{y} to produce S units of \boldsymbol{x}
 - $\bullet \ c'(S)>0 \ \text{and} \ c''(S)\geq 0$
 - firm maximizes profit: pS c(S)
 - supply function for good x is implicitly defined by the marginal condition $(MR=MC), \ p=c^\prime(S(p))$
- Equilibrium: $\underbrace{S(p)}_{\text{Supply}} = \underbrace{D(p+\tau)}_{\text{Demand}}$

Marshallian Surplus (3)

- Consider the introduction of a small tax: $d\tau > 0$ (see figure)
 - On the graph, we see cons surplus, producer surplus, tax revenue, and DWL
 - DWL is what is lost over and above what is collected in taxes. This the triangular area in the picture.



Marshallian Surplus (4)

- There are at least two ways of measuring the area of the triangle (the size of the DWL/EB):
 - 1. In terms of supply and demand elasticities
 - 2. In terms of the total change in equilibrium quantity caused by the tax

Marshallian Surplus (5)

• <u>Method 1</u>: Measuring EB in terms of supply and demand elasticities:

$$EB = \left(\frac{1}{2}\right) dQd\tau$$

$$EB = \left(\frac{1}{2}\right) S'(p)dpd\tau = \left(\frac{1}{2}\right) \left(\frac{pS'}{S}\right) \left(\frac{S}{p}\right) \left(\frac{\eta_D}{\eta_S - \eta_D}\right) d\tau^2$$

$$EB = \frac{1}{2} \left(\frac{\eta_S \eta_D}{\eta_S - \eta_D}\right) (pQ) \left(\frac{d\tau}{p}\right)^2$$

- 2nd line uses incidence formula, $dp = \left(\frac{\eta_D}{\eta_S \eta_D}\right) d\tau$
- 3rd line uses definition of η_S
- 3rd line shows common intuition that EB increases with the square of the tax and with elasticities of S and D

Marshallian Surplus (6)

- <u>Method 1</u>: Measuring EB in terms of supply and demand elasticities (cont'd):
- Tax revenue $R = Qd\tau$, so useful expression is deadweight burden per dollar of tax revenue:

$$\frac{EB}{R} = \frac{1}{2} \frac{\eta_S \eta_D}{\eta_S - \eta_D} \frac{d\tau}{p}$$

Marshallian Surplus (7)

- <u>Method 2</u>: Measuring EB in terms of total change in equilibrium quantity caused by tax:
- Define $\eta_Q = -\frac{dQ}{d\tau} \frac{P}{Q}$ as the effect of a 1% increase in the initial price via a tax change on equilibrium quantity (elasticity version of incidence formula)
- Then defining EB using change in quantity and change in price:

$$EB = -\left(\frac{1}{2}\right) dQd\tau$$
$$= -\left(\frac{1}{2}\right) \frac{dQ}{d\tau} \left(\frac{p}{Q}\right) \left(\frac{Q}{p}\right) d\tau d\tau$$
$$= \left(\frac{1}{2}\right) \eta_Q(pQ) \left(\frac{d\tau}{p}\right)^2$$

- Again, the EB is a function of the square of the tax and the sensitivity to price changes, η_Q

Marshallian Surplus (8)

- Key Result 1: Deadweight burden is increasing at the rate of the square of the tax change and deadweight burden over tax revenue increases linearly with the change in tax.
- Key Result 2: Deadweight burden increases with the absolute value of the elasticities (note that if *either* elasticity is zero, there is *no* DWB)

Marshallian Surplus (9)

- Important consequence: With many goods, the most efficient (efficient = keeping DWB as low as possible) way to raise tax revenue is:
 - Tax relatively more the inelastic goods. e.g., medical drugs, food. But what's the tradeoff?
 - Spread the taxes across all goods so as to keep tax rates relatively low on all goods (because DWB increases with the square of the tax rate)
 - Efficient tax systems spread the burden broadly. Thus, efficient tax systems have a broad base and low rates.
- The fact that DWL rises with the square of the tax rate also implies that government should not raise and lower taxes, but rather set a long-run tax rate that will meet its budget needs on average.
 - For example, to finance a war, it is more efficient to raise rates by a small amount for many years, rather than a large amount for one year (and run deficits in the short-run).
 - This notion can be thought of as "tax smoothing," similar to the notion of individual consumption smoothing.

Which elasticity? Marshallian or Hicksian

- Since the DWL depends on the elasticities of supply and demand, we need to know which elasticities to use.
- The discussion above used the Marshallian, or uncompensated, elasticity measures to measure the DWL.
- As discussed in micro theory, this is not the ideal measure of consumer surplus.
 - The appropriate elasticity is the Hicksian compensated elasticity.
 - We just want the substitution effect
 - This will allow us to consider consumer utility more directly
 - Hicksian is usually not used because it is difficult to measure empirically.
- There is a further problem in the context of analyzing tax policy:
 - The measure of excess burden for a particular tax depends on the order in which they are imposed.
 - That is, there is a "path dependence" problem.

Hicksian Compensated Demand

• The indirect utility function, V(p, m), links maximized utility to prices (p) and income (m):

$$V(p,m) = \max_{\{x_1,x_2\}} U(x_1,x_2)$$

subject to the budget constraint:

$$q_1x_1 + q_2x_2 = m$$

- The solution is the Marshallian or uncompensated demand function: $x_i(p,m)$
- The expenditure function is defined as the minimum expenditure needed at given prices to generate a given level of utility:

$$E(p,\bar{U}) = \min_{\{x_1,x_2\}} \sum_{i=1}^{2} p_i x_i$$

subject to: $U(x_1, x_2) = \overline{U}$

- The solution to the expenditure minimization problem is the Hicksian or compensated demand function, $x_i^c(p,\bar{U})$
 - It is "compensated" because income is adjusted so that utility at the given prices equals the utility at the previous price and income.
 - Thus the consumer is compensated for the income effect of the price changes

DWL with Compensated Demand (2)

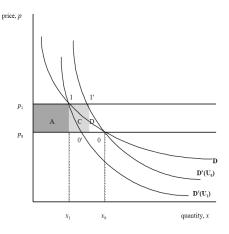
- Compensated demands naturally lead to the use of the expenditure function to define the DWL.
- Excess burden is the is the amount, in excess of revenue raised, that consumers must be compensated to maintain utility in the face of a tax induced price change.

$$DWL = CV - R = E(p_1, U_0) - E(p_0, U_0) - R$$

- This measure is well defined regardless of the order of evaluation.
 - There is no path dependence problem.
- Note that graphically, CV is the area to the left of the compensated demand curve

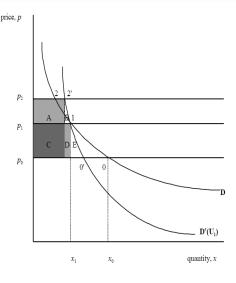
DWL with Compensated Demand

- Rather than the uncompensated demand, *D*, use the compensated demand D^C .
- If we had a same tax increase, p₀ to p₁, but held utility constant, the DWL would be D and revenue would be A + C.



DWL with Preexisting Distortions (taxes)

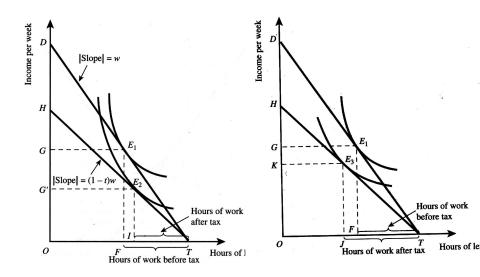
- Now an additional tax raises the price to p_2 .
- Change in tax revenue is: A D.
- Marginal excess burden, the DWL, from the 2nd tax, is the "Harberger triangle," B + D.
 - Note that while the triangle will approach zero for small tax change the rectangle might not. Hence the importance of "preexisting distortions."



Excess Burden of Income Taxation

- The lessons learned from previous analysis of the impact of excise or commodity taxes also applies to the taxation of labor.
- Recall the labor-leisure choice problem where consumers value the consumption of goods and leisure.
- Taxing labor income "flattens" the budget constraint leading to substitution and income effects.
- Taxing income would lead to greater consumption of leisure via the substitution effect but the income effect goes in the opposite direction. (assuming leisure is a normal good)
- It is not clear whether taxing income increases or decreases leisure consumption and hence work.

Excess Burden of Income Taxation (2)



Cobb-Douglas Labor Supply

Note what happens if we choose C-D utility

• Suppose that utility is of the form

$$U = c^{\alpha} H^{\beta}$$

• The budget constraint is

$$c = wL + Z$$

• and the time constraint is

$$L + H = 1$$

• Note that we have set ("normalized") the maximum work time to 1 hour for convenience

Cobb-Douglas Labor Supply (2)

• The Lagrangian expression for utility maximization is:

$$\mathscr{L} = c^{\alpha} H^{\beta} + \lambda (w + Z - wH - c)$$

• First-order conditions are

1.
$$\frac{\partial \mathscr{L}}{\partial c} = \alpha c^{\alpha - 1} H^{\beta} - \lambda = 0$$

2.
$$\frac{\partial \mathscr{L}}{\partial L} = \beta c^{\alpha} H^{\beta - 1} - \lambda w = 0$$

3.
$$\frac{\partial \mathscr{L}}{\partial \lambda} = w + Z - wH - c = 0$$

Cobb-Douglas Labor Supply (3)

Dividing the first by the second yields

$$\frac{\alpha H}{\beta c} = \frac{1}{w}$$

which means that:

$$wH = \frac{\beta}{\alpha}c$$

Cobb-Douglas Labor Supply (4)

• Substitution into the full income constraint yields

$$c = \frac{\alpha}{(\alpha + \beta)}(w + Z)$$

$$H = \left(\frac{\beta}{(\alpha + \beta)}(w + Z)\right)/w$$

- the person spends $\frac{\alpha}{\alpha+\beta}$ of his income on consumption and $\frac{\beta}{\alpha+\beta} = 1 \frac{\alpha}{\alpha+\beta}$ on leisure (if there are CRS (i.e. $\alpha + \beta = 1$), then there fractions are α and 1α)
- the labor supply function is: $L(w,Z) = 1 H = 1 \frac{\beta(w+Z)}{(\alpha+\beta)w}$

Cobb-Douglas Labor Supply (5)

If CRS (i.e. $\alpha + \beta = 1$) then:

- If Z=0, the person will work $(1-\beta)$ of each hour no matter what the wage is
 - the substitution and income effects of a change in w offset each other and leave H unaffected

• If
$$Z > 0 \Rightarrow \frac{\partial L}{\partial w} = \frac{\beta Z}{w^2} > 0$$

- the individual will always choose to spend βZ on leisure
- Since leisure costs w per hour, an increase in w means that less leisure can be bought with ${\cal Z}$
- Note that $\frac{\partial L}{\partial Z} < 0$
 - an increase in non-labor income allows this person to buy more leisure
 - income transfer programs are likely to reduce labor supply
 - lump-sum taxes will increase labor supply

Cobb-Douglas Labor Supply (6)

- Is Cobb-Douglas realistic? It depends
- Recall Cobb-Douglas utility function have the property that the substitution effect and the income effect cancel each other out (if there is no non-labor income).
- For some workers this may not be a troubling approximation.
 - Some workers will work full time at whatever the going rate is.
- But clearly this is not a general case.

Labor Supply More Generally

- Suppose that utility is of the general form, U(c, H) and the budget constraint in the presence of taxation is c = (1-t)wL + (1-t)Z
- Utility maximization will provide a labor supply function, $L(t,w,{\boldsymbol Z})$
- Increasing t has 3 effects
 - 1. It reduces the after-tax non-wage income; if leisure is a normal good, this increases labor supply
 - 2. It also generates an income effect in reducing after-tax wage income which increases labor supply
 - 3. There is also a substitution effect since the price of leisure falls relative to other consumption thus decreasing labor supply

Labor Supply More Generally (2)

- Note that the two income effects will depend on the average tax rate while the substitution effect will depend on the marginal tax rate.
 - For small tax changes this distinction is not important.
- For labor supply the impact of the tax change is:

$$\frac{\partial L}{\partial t} = \frac{\partial L}{\partial P_L} \frac{\partial P_L}{\partial t} + \frac{\partial L}{\partial M} \frac{\partial M}{\partial t}$$

• Where M is non-labor income (after tax): M = (1 - t)Z, and P_L is the after-tax price of labor: $P_L = (1 - t)w$

Labor Supply More Generally (3)

• From the Slutsky equation we can evaluate the change in labor as the price of labor changes:

$$\frac{\partial L}{\partial P_L} = \frac{\partial L}{\partial P_L}|_U + L\frac{\partial L}{\partial M}$$

• Thus the total impact of a tax change is:

$$\frac{\partial L}{\partial t} = -w \frac{\partial L}{\partial P_L} |_U - (wL + Z) \frac{\partial L}{\partial M}$$

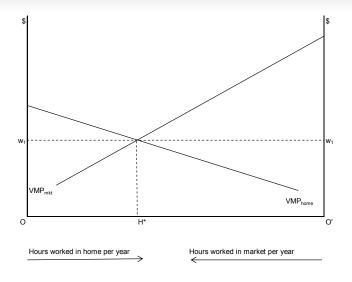
Labor Supply More Generally (4)

- The 1st term is the substitution effect, which is negative.
- The latter term consists of the two income effects. This term will be positive if leisure is a normal good.
 - Note that the derivative multiplied by income, thus we would expect that this term is larger the greater is income.
 - This implies that the rich should be less sensitive to taxes than the poor. Is this what we expect?
- The issue is that we have greatly simplified the labor market in a manner that may impact the result.
 - In reality, labor supply isn't very flexible, it is more of a binary choice: Either you are in the labor market or you are not.

Differential Taxation of Labor

- Another implication of the taxation of labor is the distortion between labor market production and household production.
- Assume individuals allocate their time between housework and market work to maximize their total incomes.
- In equilibrium, the value of the marginal product (VMP) of labor for each sector is equal. VMP_L exhibits diminishing marginal returns (as more hours are spent in each sector, the incremental value of each additional hour decreases), hence there is a downward slope.

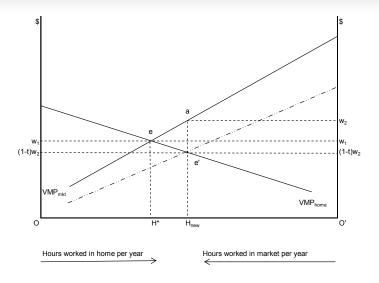
Differential Taxation of Labor (2)



Differential Taxation of Labor (3)

- Assume an ad valorem tax, t, is levied on labor market income.
- This moves the labor market VMP downward by a percentage t.
- At H^* , $VMP_{home} > (1-t)VMP_{mkt}$.
- Hours spent in home production increases, as individuals reallocate their labor/home production hours.
- This moves the equilibrium point to the right, until $VMP_{home} = (1 t)VMP_{mkt}$.
- The new equilibrium occurs at H_{new} and $(1-t)w_2$.

Differential Taxation of Labor (4)



Differential Taxation of Labor (5)

- The increased allocation to the household sector is inefficient:
 - at H_{new} , $VMP_{mkt} > VMP_{home}$.
 - But there is no incentive to reallocate, because people care more about after-tax income than pre-tax income.
 - The resulting decrease in real income is the excess burden of the tax.
- To measure the excess burden, take the area of eae', which is the difference between the loss in labor production and the increase in household production. This area is $0.5(H^* H_{new})(tw_2)$.

The Impact of Taxation on Savings

- Consider a consumer that lives two periods and only works in the first at a wage of w.
 - Labor supply is assumed to be inelastic (hence *w* is exogenous)
 - One might consider this a simply model of pre and post retirement behavior
- Earnings less consumption in the first period, C_1 , is saved at interest rate r.
- The interest earnings are taxed in the second period at rate t the remainder providing consumption in the second period, C_2 .
- The consumer has a utility function of the form $U(C_1, C_2)$.

The Impact of Taxation on Savings (2)

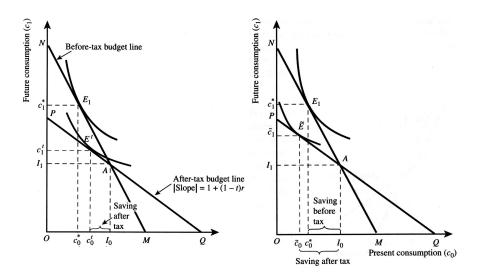
- The intertemporal budget constraint for this consumer is: $C_1 + pC_2 = w$ where p is the relative price of consumption in period 2 versus period 1, $p = \frac{1}{[1+(1-t)r]}$
- Let $M = \frac{w}{p}$ be value of lifetime after-tax income in the second period.
- Changes in taxation change the relative price of consumption.
- This price change will generate income and substitution effects.

$$\frac{\partial C_1}{\partial p} = \frac{\partial C_1}{\partial p}|_U - \frac{\partial C_1}{\partial M}\frac{\partial M}{\partial p}$$

The Impact of Taxation on Savings (3)

- The first term is the substitution effect which is positive.
 - As the price of consumption in period 2 increases relative to period 1, consumption in period 1 increases and savings decrease
- The second term is the income effect which is negative.
 - As *p* increases, lifetime income falls, so consumption in both periods falls.
- Hence the overall impact is unclear. A price, or tax change, may either increase or decrease savings.

The Impact of Taxation on Savings (4)



The Impact of Taxation on Savings (5): Substitution effect

• The critical element of the preference function is the intertemporal elasticity of substitution:

$$\sigma = \frac{\partial log(C_1/C_2)}{\partial log \ p}|_U = \frac{\partial log(C_1)}{\partial log \ p}|_U - \frac{\partial log(C_2)}{\partial log \ p}|_U$$

• From the intertemporal budget constraint:

$$C_1(p, U) + pC_2(p, U) = E(p, U)$$

• Since the compensated demands are the derivatives of the expenditure function:

$$\frac{\partial C_1}{\partial p}|_U + p\frac{\partial C_2}{\partial p}|_U = 0 \Rightarrow \frac{1}{C_2}\frac{\partial C_1}{\partial p}|_U = -\frac{p}{C_2}\frac{\partial C_2}{\partial p}|_U = -\frac{\partial \log(C_2)}{\partial \log p}|_U$$

The Impact of Taxation on Savings (6): Substitution effect

• Substituting this into the elasticity of substitution:

$$\sigma = \frac{\partial log(C_1)}{\partial log \ p}|_U + \frac{1}{C_2}\frac{\partial C_1}{\partial p}|_U = \frac{w}{pC_2}\frac{\partial log(C_1)}{\partial log \ p}|_U$$

• Then rearranging yields

$$\frac{\partial log(C_1)}{\partial log \ p}|_U = \sigma \frac{pC_2}{w}$$

• Holding utility constant the elasticity of 1st period consumption with respect to price (closely related to the substitution effect) is a function of the intertemporal elasticity of substitution and the savings rate

The Impact of Taxation on Savings (7): Income effect

• Note that the derivative in the income effect can be expressed as: $\partial C_1 = C_1 \partial \log C_1 = C_1$

$$\frac{\partial C_1}{\partial w} = \frac{C_1}{w} \frac{\partial \log C_1}{\partial \log w} = \frac{C_1}{w} \eta$$

- Where η is the income elasticity of 1st period consumption.
- Now using an elasticity measure, the income and substitution effects can be expressed as:

$$\frac{\partial log \ C_1}{\partial log \ p} = \frac{pC_2}{w}\sigma - \frac{pC_2}{C_1}\eta \frac{C_1}{w} = \frac{pC_2}{w}(\sigma - \eta)$$

The Impact of Taxation on Savings (8)

- So to get a significant impact of taxation on savings the intertemporal elasticity of substitution has to be relatively large.
- Note that the permanent income hypothesis implies that $\eta = 1$ (i.e., consumption moves 1-for-1 with lifetime income).
- There is not a consensus on these parameters often very different in macro vs. micro studies

Example of Taxation of Savings with Cobb-Douglas

• Example:

- Individual lives two periods
- Utility functions: $U = C_1^{0.5} C_2^{0.5}$
- Wage w in period 1, no income in 2nd period
- Tax rate of t
- Real interest rate of r
- We know the solution will have equal expenditures on consumption in each period due to the symmetric Cobb-Douglas utility function.

Example of Taxation of Savings with Cobb-Douglas (2)

•
$$\mathscr{L} = C_1^{0.5} C_2^{0.5} + \lambda (w - C_1 - pC_2)$$

• Where, $p = \frac{1}{1 + (1 - t)r}$

FOCs:

•
$$\frac{\partial \mathscr{L}}{\partial C_1}$$
 : $0.5C_1^{-0.5}C_2^{0.5} = \lambda$
• $\frac{\partial \mathscr{L}}{\partial C_1}$: $0.5C_1^{0.5}C_2^{-0.5} = p\lambda$

- $\Rightarrow C_1 = pC_2$
- $\Rightarrow C_1 = \frac{w}{2}$

•
$$\Rightarrow C_2 = \frac{w}{2p}$$

• Thus as $t\uparrow$, $C_2\downarrow$ and $U\downarrow$

Example of Taxation of Savings with Cobb-Douglas (3)

- The taxation of savings has distorted the choice of consumption between periods by altering the relative price.
- This generates a deadweight loss.
- This is the primary motivation for many arguments for the superiority of consumption taxation over income taxation.
- General notes on taxing savings:
 - A consumption tax is equivalent to a tax on income less savings
 - Taxing savings is equivalent to taxing consumption at an increasing rate over time
 - Dynamic efficiencies are important since savings=investment

Choice between taxation of labor or of savings

- Assume a simple 2 period life cycle model
- The individual has a log-linear utility function

$$U = ln(C_1) + ln(C_2) + ln(L)$$

• where C_1 and C_2 are consumption during the two periods of life and L is the leisure during the first period. The individual is assumed to be fully retired in the second period.

Choice between taxation of labor or of savings (2)

• The individual's lifetime budget constraint is

$$[w(1-t)(1-L) - C_1][1 + (1-T)r] = C_2$$

- where
 - t is the labor income tax
 - T is the tax on investment income
 - r is the pretax real rate of return on savings.
- Substituting this expression for C_2 into the utility function and maximizing utility with respect to L and C_1 implies first order conditions:
 - L = 1/3
 - $C_1 = w(1-t)/3$
 - $C_2 = w(1-t)[1+(1-T)r]/3$

Choice between taxation of labor or of savings (3)

• Substituting these expressions into the utility function and simplifying implies

$$u = -3ln(3) + 2ln[w(1-t)] + ln[1 + (1-T)r].$$

• The revenue from a labor income tax is

$$TAX_L = tw(1-L) = 2tw/3.$$

• The revenue from a capital income tax, collected in the second period, is

$$TAX_{CAP} = Tr[w(1-t)(1-L) - C1]$$

• If there is no labor income tax, this simplifies to

$$TAX_{CAP} = Tr[w(1-L) - C1] = Trw/3$$

• The present value of this tax as of the first period is

$$TAX_{CAP}/(1+r) = Trw/3(1+r).$$

Choice between taxation of labor or of savings (4)

- Now we want to compare these two results, so we normalize the tax revenue
- Choose T such that $TAX_{CAP}/(1+r) = TAX_L$
- This implies: Trw/3(1+r) = 2tw/3

• or:
$$T = 2t(1+r)/r$$

Choice between taxation of labor or of savings (5)

• The utility expression derived above:

$$U = -3ln(3) + 2ln[w(1-t)] + ln[1 + (1-T)r]$$

• implies that with a pure labor income tax (i.e., if T = 0) the utility level is

$$U_L = -3ln(3) + 2ln(w) + 2ln(1-t) + ln[1+r]$$

• With no labor income tax but a capital tax that produces the same present value of revenue (i.e., with T = 2t(1+r)/r), the utility level is:

$$U_{CAP} = -3ln(3) + 2ln(w) + ln(1+r) + ln(1-2t)$$

Choice between taxation of labor or of savings (6)

 The utility level with the pure labor income tax exceeds the utility level with the pure capital income tax, U_L > U_{CAP}, if

$$2ln(1-t) > ln(1-2t)$$

$$ln(1-t)2 > ln(1-2t)$$

$$(1-t)2 > (1-2t)$$

 But this is true for any t ≥ 0, showing that for this loglinear utility case the labor tax produces higher utility than a capital tax with equal present value of revenue.

Conclusion: Taxation and Economic Efficiency

- In general broad tax bases and low rates reduce excess burden and increase efficiency
- In general the taxation of savings (which is in effect a tax on future consumption) generates an excess burden
- Keep in mind that there is generally a tradeoff between equity and efficiency
 - The choice will be driven by the social welfare function assumed